

Virtual storage plant offering strategy in the day-ahead electricity market

Kristina Pandžić^a, Hrvoje Pandžić^{b,*}, Igor Kuzle^b

^a Croatian Transmission System Operator – HOPS Ltd., Croatia

^b Department of Energy and Power Systems, Faculty of Electrical Engineering and Computing, University of Zagreb, Croatia



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ABSTRACT

Energy storage is gaining an important role in modern power systems with high share of renewable energy sources. Specifically, large-scale battery storage units (BSUs) are an attractive solution due to their modularity, fast response and ongoing cost reduction.

This paper aims to formulate, analyze and clarify the role of merchant-owned BSUs in the day-ahead electricity market. It defines virtual storage plant (VSP) as a set of BSUs distributed across the network. A VSP offering model is formulated as a bilevel program in which the upper-level problem represents the VSP profit maximization and operation, while the lower-level problem simulates market clearing and price formation. This mathematical problem with equilibrium constraints (MPEC) is converted into a mixed-integer linear program (MILP). This is afterwards expanded to a game of multiple VSPs formulating an equilibrium problem with equilibrium constraints (EPEC), which is solved using the diagonalization procedure.

The proposed model is applied to an updated IEEE RTS-96 system. We evaluate the impact VSPs have on the locational marginal prices and compare the coordinated approach (all BSUs operated under a single VSP), i.e. the MPEC formulation, to the competitive approach (multiple VSPs competing for profit), i.e. the EPEC formulation.

1. Introduction

1.1. Motivation

The increasing share of renewable energy sources (RES) is changing the paradigm of modern power systems. The term power itself indicates a constant balance between demand and supply. However, an increased share of non-controllable RES, i.e. solar and wind, results in less dispatchable capacity at the disposal to the system operator. Thus, the technical ability to meet the uncertain net demand is reducing because RES output can vary within a market interval [1]. Many studies report that intermittent non-dispatchable RES increase reserve requirement, e.g. Italian historical data analyzed in [2] report a decrease of energy prices and increase of reserve costs as a result of the RES integration. These technical and economic conditions make large-scale energy storage solutions attractive, as they enable switching to the energy system paradigm, as opposed to the power system paradigm. As opposed to the current power system paradigm, where generation and demand need to be balanced at each point in time, in an energy system, generation and demand need to be balanced over a longer time period, e.g. hours, while energy storage acts as a buffer that voids the short-term generation-load imbalances. In other words, energy storage enables secure

and stable power system operation even without the constant generation-demand balance since it acts as a generation and demand asset interchangeably. Rassmussen et al. [3] claim that large distributed energy storage would enable covering the entire electricity demand in Europe using only RES. Related to this, electricity generation of wind turbines is already reaching high levels. In 2015, Danish wind turbines generated an equivalent of 42 percent of the overall electricity demand in that country [4].

Regulative authorities have not yet issued clear regulating mechanisms governing the use of energy storage in electricity markets. Joint European Association for Storage of Energy and European Energy Research Alliance recommendations for European Energy Storage Technology Development Roadmap towards 2030 [5] recognizes that energy storage technology can be used to provide regulated services to system operators and non-regulated services in electricity markets (see the model presented in [6]). In the USA, Federal Energy Regulatory Commission (FERC) has issued orders to help facilitate energy storage in regulated markets. FERC Order 555 issued a pay-per-performance incentive for resources that can provide quicker and more precise responses to frequency regulation signals. This enables energy storage technologies which outperform the conventional regulation providers, such as gas- and coal-fired power plants, to receive higher

* Corresponding author.

E-mail addresses: kristina.pandzic@hops.hr (K. Pandžić), hrvoje.pandzic@fer.hr (H. Pandžić), igor.kuzle@fer.hr (I. Kuzle).

Nomenclature

Sets

Ω^B	set of piecewise linear segments of each generating unit's offer curve, indexed by b .
Ω^C	set of piecewise linear segments of each bus' demand bid curve, indexed by c .
Ω^H	set of BSUs, indexed by h .
Ω^I	set of generating units, indexed by i .
Ω^J	set of VSP owners, indexed by j .
Ω^L	set of transmission lines, indexed by l .
Ω^S	set of buses, indexed by s .
Ω^T	set of hours, indexed by t .
Ω^W	set of wind farms, indexed by w .

Binary variables

$x_{t,h}^{\text{ch}}$	BSU charging status (1 if BSU h is charging during hour t , 0 otherwise).
$x_{t,h}^{\text{dis}}$	BSU discharging status (1 if BSU h is discharging during hour t , 0 otherwise).

Continuous variables

$d_{t,s,c}$	power consumption on segment c at s during hour t (MW).
$g_{t,i,b}$	power output on segment b of generator i during hour t (MW).
$k_{t,w}$	power output of wind farm w during hour t (MW).

$p_{l,s,m}^f$	power flow through line s – m during hour t (MW).
$q_{t,h}^{\text{ch}}$	power purchased by BSU h during hour t (MW).
$q_{t,h}^{\text{dis}}$	power sold by BSU h during hour t (MW).
$soe_{t,h}$	state of energy of BSU h during hour t (MWh).
$\alpha_{t,h}^{\text{ch}}$	charging bid of BSU h during hour t (MW).
$\alpha_{t,h}^{\text{dis}}$	discharging offer of BSU h during hour t (MW).
$\theta_{t,s}$	voltage angle of bus s during hour t (rad).
$\lambda_{t,s(h)}$	locational marginal price at bus s where BSU h is located (\$/MW).

Parameters

ch_h^{max}	charging capacity of BSU h (MW).
dis_h^{max}	discharging capacity of BSU h (MW).
$d_{t,s,c}^{\text{max}}$	capacity of demand block c at bus s during hour t (MW).
η_h^{ch}	charging efficiency of BSU h .
η_h^{dis}	discharging efficiency of BSU h .
$g_{i,b}^{\text{max}}$	capacity of offering block b of generator i (MW).
$k_{t,w}^{\text{max}}$	available wind generation of wind farm w (MW).
λ_h^{ch}	bidding price of BSU h (\$/MW).
λ_h^{dis}	offering price of BSU h (\$/MW).
$\lambda_{s,c}^D$	bidding price of demand block c at bus s (\$/MW).
$\lambda_{i,b}^G$	offering price of block b of generator i (\$/MW).
$p_{s,m}^{\text{max}}$	transmission capacity of line s – m (MW).
soe_h^{max}	energy capacity of BSU h (MWh).
soe_h^{min}	minimum energy stored in BSU h (MWh).
su_{sm}	susceptance of line connecting nodes s and m (S).

remuneration. An evaluation of the utility of energy storage for different market paradigms and ownership models is available in [7].

The main disadvantages of conventional large-scale energy storage, i.e. pumped hydro and compressed air energy storage, are geographical constraints and bulkiness. Due to these limitations, conventional storage technologies are less suitable than modular storage devices that can be installed at virtually any location without a significant ecological footprint. A review of the current state of energy storage technologies indicates that batteries are generally a versatile energy storage technology that can be installed at almost any location [8]. A common grid-scale battery technology today is lithium-ion, which is suitable for providing frequency regulation [9]. Energy-to-power ratio of lithium-ion battery installations is usually lower than 1 and installed capacities are much lower than the ones of traditional energy storage, i.e. pumped hydro [10]. On the other hand, NaS batteries are more suitable for congestion relief as their energy-to-power ratio is 7 [11]. On top of this, the cost of batteries has been reducing due to their use in electric vehicles [12]. A review on battery energy storage technologies is available in [13].

Large-scale use of battery storage has a wide range of applications, providing different values to the power system. Battery storage units (BSUs) can help in peak shaving [14] and increasing the system flexibility and reliability providing power regulation services [15]. Fast-response energy storage, such as BSU, has the potential to replace fast-ramping generation resources [16]. Economics of transmission or capacity investment deferral are addressed in [17]. Therefore, the development of energy storage technology, especially battery technology, might offer solutions for many critical challenges in smart grids [18,19]. Combining these applications reduces the payback period making the investment more attractive.

Storage operation highly depends on its ownership. For instance, Terna's BSUs are used to ensure safety and cost-effective management of the Italian transmission grid [9]. In a vertically integrated utility, BSUs are used to reduce the overall operating cost [20]. Finally,

merchant-owned BSU is operated in a way to maximize its profit [21]. In case multiple BSUs are operated by different owners, they compete with each other to make profit. This resembles an equilibrium problem with equilibrium constraints (EPEC), i.e. a multiple-leader-common-follower game, as introduced in [22]. EPEC structure is particularly common in the analysis of deregulated electricity markets [23,24], where players maximize their benefit in the form of mathematical problems with equilibrium constraints, MPECs, e.g. [25], while adhering to the same market-clearing rules. For instance, in [26] an EPEC model is derived to find equilibria reached by strategic producers in a pool-based transmission-constrained electricity market using KKT conditions, while in [27] the authors use diagonalization method to find multiple equilibria of generator maintenance schedules in electricity market environment.

The goal of the presented model is to formulate, model and analyze storage operation in the day-ahead electricity market. A VSP owns and operates its BSUs distributed across the system in order to maximize their overall market performance. It derives an optimal strategy centrally and sends control signals to all its BSUs to charge/discharge.

1.2. Literature review

Generally, integration of energy storage in power systems can be observed either from the system-wide perspective or the merchant perspective. The system-wide perspective is usually modeled as a unit commitment model whose goal is to minimize overall system operating costs, regardless on the profit an energy storage is making. An exception in the literature is [28], which minimizes the overall system costs while ensuring the profitability of a merchant-owned energy storage. On the other hand, there are models which take perspective of a storage owner, thus aiming at maximizing the profit a storage is making in electricity markets. In these models, energy storage can be a significant market player able to affect the market prices, i.e. price maker models, or its

capacity can be relatively low as compared to other generating and demand capacities, which means the storage is a price taker.

1.2.1. System-wide studies

The authors of [29] assess the potential of energy storage to economically decrease wind curtailment and/or system costs. The authors report that batteries with higher power rating result in less wind curtailment, but also require lower installation costs. The sensitivity analysis indicates that the most relevant parameters are the existence of subsidies, the installation cost of transmission lines, battery degradation and life cycle duration.

An approximate unit commitment model based on load duration curve is presented in [30]. System states framework is used to preserve the storage intertemporal dependencies. Since the same system states might have different storage states of energy, which depend on the states before and after the current time period, the authors use the difference in state of energy, and not stored energy, as variables. The proposed method improves computational tractability by 90% as compared to the chronological hour-by-hour models, while causing an error of less than 2%.

Paper [31] uses stochastic programming models to derive optimal unit commitment policy with pump storage plants as an important storage technology in which the investments are expected to increase.

In [20], the authors present a near-optimal three-stage technique for siting and sizing of battery energy storage in transmission network. The authors conclude that optimal storage locations are near wind farms or along the congested corridors of a network.

A framework for storage portfolio optimization in transmission-constrained power networks is proposed in [32]. The model optimizes storage operation and siting given a fixed technology portfolio. Additionally, the authors optimize the portfolio itself, thus demonstrating the importance of choosing a proper technology.

Paper [33] presents a study on utilizing energy storage to manage intra-hour variability of the net load. A standard unit commitment model is implemented in PLEXOS, with addition of primary, secondary and tertiary reserve provision. Implementation of the model to the expected plant portfolio in Irish system in 2025 reveals that integration of storage should lead to 15% less cycling of conventional units and up to 40% savings in operating costs.

1.2.2. Price-taking merchant-owned storage

Merchant-owned energy storage can be used to support the local renewable generation or independently act in electricity markets. In [34] the authors propose a joint bidding mechanism of a wind farm and a pumped hydro plant in the day-ahead and ancillary service markets. A profit maximization model of a virtual power plant that includes energy storage is proposed in [35]. This model accounts for bilateral contracts while maximizing the profit in the day-ahead electricity market. A similar study [36] shows that using batteries to compensate for intermittent and variable generation results in a smoother overall generation curve.

As opposed to [34–36], where energy storage is used to support renewable generation, the authors in [37] present a profit maximization model for a price-taker storage unit that participates in energy and reserve day-ahead market and energy hour-ahead market. Uncertain parameters are the price of power and reserve in the hour-ahead market, and the actual reserve utilization. The values for these parameters are obtained by presolving a stochastic unit commitment model for different realizations of wind. The authors demonstrate the importance of considering the uncertainty of market prices (due to uncertain nature of wind) in the presented independent energy storage profit maximization model. Paper [38] also deals with energy storage economics when participating in arbitrage and regulation services within different markets. The case study results show that high potential revenues could be generated from ancillary services market. A backwards induction approach is employed in [39] to derive optimal

bidding strategy of a battery storage operator. The authors consider the storage device exhaustable with a limited number of cycles and lifetime.

The authors of [40] present a case study to demonstrate that storage technologies may have competitive advantage over the peaking generators, due to the ability to earn revenue outside of extreme peak events. The main driver for storage options in an energy-only electricity market is extreme prices, which in turn is dependent on capacity requirements.

Paper [41] uses a portfolio of energy trade strategies to determine the value of arbitrage for energy storage across the European markets. The results show that arbitrage opportunities exist in less integrated markets, characterized by significant reliance on energy imports and lower level of market competitiveness.

The authors of [42] provide a comprehensive stochastic energy storage valuation framework which allows a storage system to provide multiple services simultaneously, i.e., the frequency regulation service and energy shifting service are co-optimized in the market operations. An operational optimization model is developed to determine the storage system's optimal dispatch sequences with a frequency regulation service price forecasting model. Simulation results show that the majority of the revenue comes from regulation services.

1.2.3. Price-making merchant-owned storage

In [21], the authors assess the impact of strategic energy storage behavior in a nodal electricity market. The results indicate that the storage profit maximization goal is not always in line with the social welfare improvement. Namely, the storage aims to retain the price volatility among hours in order to maximize its profit.

A bilevel formulation of a coordinated scheduling of multiple storage units is presented in [43]. The upper-level problem sets optimal market bids and offers for each storage unit, while the lower-level problem simulates market clearing procedure. The initial formulation is converted to MPEC and linearized using the Karush–Kuhn–Tucker (KKT) conditions of the lower-level problem. The paper also contains stochastic and basic robust (maximizing the profit of the least profitable scenario) reformulations of the upper-level problem. The important conclusions are that the transmission congestion is usually beneficial for energy storage profit and that locations of individual storage units affect their coordinated scheduling and profit.

The authors of [44] propose a bilevel formulation where the upper-level problem seeks to maximize merchant storage arbitrage profit, while the lower-level problem simulates market clearing. The lower-level problem is a stochastic problem, depending on wind scenarios. The paper specifically analyzes the impact of the thermal generator flexibility on storage profit. The authors illustrate how storage takes advantage of different system conditions. The case study shows that a congested transmission grid enables storage to exercise spatio-temporal arbitrage, bringing much more revenue as compared to only exploiting limited ramping constraints of conventional generators.

Paper [45] examines the impact storage has on generator company profit increment due to strategic bidding. The authors exploit the bilevel structure, where the upper-level problem maximizes the total profit of generating companies. The lower-level problem is a simulation of transmission-unconstrained market clearing procedure. The conducted case study indicates that energy storage diminishes the possibility of generating companies to exercise market power at high-demand hours, but increases it at off-peak periods. However, the reduction at peak periods is higher than the increase in off-peak periods due to the larger slope of the strategic marginal cost curve. Therefore, energy storage is beneficial to the bidding side, as it helps preserving the bidding side surplus.

The authors of [46] propose a multi-period equilibrium problem with equilibrium constraints to study strategic behavior of various generators. The model considers energy storage systems as price makers in the energy market and their impact on the market equilibrium is

thoroughly analyzed. The nonlinear complementarity constraints are handled using a reformulation technique. The paper [47] proposes a bilevel equilibrium model to study market equilibrium interactions between energy storage and wind and conventional generators.

Paper [48] demonstrates that in some market structures energy storage can reduce social welfare, which contradicts conventional opinion of reducing the welfare losses by adding firms to an imperfectly competitive market. These findings have important implications for storage development and storage-related policies.

A case study on integration of energy storage in German electricity market is presented in [49]. The results indicate that energy storage reduces price spikes and producer surplus. The authors conclude that energy storage investments are not attractive to companies that already own generation facilities.

1.3. Scope and contributions

The model proposed in this paper falls in the category of merchant-owned energy storage operation problems. While papers [21,48] are focused on energy storage impact on social welfare, [43,44] on the impact of uncertainty on energy storage operation, [44–49] on interaction between generators and energy storage, this paper aims at filling the literature gap on interaction between energy storage companies, i.e. VSPs. Specifically, we determine the benefits of a longer look-ahead horizon than a single day, the loss of profit when having independent VSPs competing in the day-ahead market and the consequences of not considering the market decisions of other VSPs when bidding in the day-ahead market.

The presented model assumes that VSP is a price maker and makes profit in the day-ahead market by performing arbitrage. We employ bilevel programming in order to model the relationship between the VSP operator's optimal bidding problem and the market operator's market clearing problem. These two problems interact in a way that the upper-level problem decides on VSPs bidding quantities (and prices), while the lower-level problem performs market clearing considering the upper-level decisions. The outcome of the lower-level problem are, among others, locational marginal prices (LMPs), which are used in the upper-level problem to calculate the VSP profit of the VSP.

We present three different models that depict different market operation of storage units. The first model is an MPEC that optimizes bidding quantities of a VSP owning a number of BSUs. It is assumed that offering and bidding prices are set to zero and to the market cap value, respectively. The second MPEC is also focused on a single-entity owned storage, but apart from the quantities, the storage operator sets the offering and bidding prices as well. Finally, the third model is an EPEC where different VSPs compete for profit by scheduling their BSUs. This EPEC is a multiple-leader-common-follower game, where different VSP owners are the leaders subject to the same follower – the market.

The contributions of the paper are summarized as follows:

1. Formulation of a VSP price and quantity offer model (MPEC) and its comparison to the quantity-only offer model.
2. Analysis and quantitative evaluation of the look-ahead horizon of the VSP operation model. Namely, even when operating a daily-cycle storage, a look-ahead horizon longer than a single day might be required.
3. Formulation of a multiple VSP model (EPEC), where a diagonalization method is utilized to evaluate possible equilibria when different storage owners compete to maximize their profit.
4. Comparison of the EPEC approach (BSUs divided among multiple VSPs) to the case of a single VSP, i.e. all BSUs operated by a single VSP.

The presented models and analysis should be interesting to energy

storage investors, operators of energy storage facilities, aggregators of distributed storage units, and balance responsible parties.

2. Formulation

This paper employs formulations to model each of the following settings:

1. Quantity-only MPEC model, i.e. offering at zero price and bidding at the market cap price, presented in Section 2.2.
2. Price-quantity MPEC model, presented in Section 2.3.
3. EPEC model, presented in Section 3.1.

The main notation used throughout the paper is listed at the beginning of the paper for a quick reference. Dual variables of the lower-level problem constraints are stated after a colon in the corresponding constraint.

2.1. Quantity-only model formulation

$$\max \sum_{t \in \Omega^T} \sum_{h \in \Omega^H} \lambda_{t,s(h)} (q_{t,h}^{\text{dis}} - q_{t,h}^{\text{ch}}) \quad (1)$$

subject to:

$$soe_{t,h} = soe_{t-1,h} + \Delta t \cdot q_{t,h}^{\text{ch}} \cdot \eta_h^{\text{ch}} - \Delta t \cdot \frac{q_{t,h}^{\text{dis}}}{\eta_h^{\text{dis}}} \quad \forall t \in \Omega^T, h \in \Omega^H \quad (2)$$

$$x_{t,h}^{\text{ch}} + x_{t,h}^{\text{dis}} \leq 1 \quad \forall t \in \Omega^T, h \in \Omega^H \quad (3)$$

$$soe_h^{\text{min}} \leq soe_{t,h} \leq soe_h^{\text{max}} \quad \forall t \in \Omega^T, h \in \Omega^H \quad (4)$$

$$\begin{aligned} \max \sum_{t \in \Omega^T} \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} \lambda_{s,c}^D \cdot d_{t,s,c} + \sum_{t \in \Omega^T} \sum_{h \in \Omega^H} \lambda_h^{\text{ch}} \cdot q_{t,h}^{\text{ch}} \\ - \sum_{t \in \Omega^T} \sum_{i \in \Omega^I} \sum_{b \in \Omega^B} \lambda_{i,b}^G \cdot g_{t,i,b} - \sum_{t \in \Omega^T} \sum_{h \in \Omega^H} \lambda_h^{\text{dis}} \cdot q_{t,h}^{\text{dis}} \end{aligned} \quad (5)$$

subject to:

$$\begin{aligned} - \sum_{w \in \Omega^W(S)} k_{t,w} - \sum_{h \in \Omega^H(S)} q_{t,h}^{\text{dis}} - \sum_{i \in \Omega^I(S)} \sum_{b \in \Omega^B} g_{t,i,b} + \sum_{l \in \Omega^L(S)} pf_{t,l}^- - \sum_{l \in \Omega^L(S)} pf_{t,l}^+ \\ = - \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} d_{t,s,c} - \sum_{h \in \Omega^H(S)} q_{t,h}^{\text{ch}} \lambda_{t,s} \quad \forall t \in \Omega^T, s \in \Omega^S \end{aligned} \quad (6)$$

$$pf_{t,l} = sus_{s,m} \cdot (\theta_{t,s} - \theta_{t,m}): \quad \beta_{t,l} \quad \forall t \in \Omega^T, l \in \Omega^L, l \in \{s, m\} \quad (7)$$

$$-pf_l^{\text{max}} \leq pf_{t,l} \leq pf_l^{\text{max}}: \quad \beta_{t,l}^{\text{min}}, \beta_{t,l}^{\text{max}} \quad \forall t \in \Omega^T, l \in \Omega^L \quad (8)$$

$$0 \leq g_{t,i,b} \leq g_{i,b}^{\text{max}}: \quad \gamma_{t,i,b}^{\text{min}}, \gamma_{t,i,b}^{\text{max}} \quad \forall t \in \Omega^T, i \in \Omega^I, b \in \Omega^B \quad (9)$$

$$0 \leq d_{t,s,c} \leq d_{t,s,c}^{\text{max}}: \quad \sigma_{t,s,c}^{\text{min}}, \sigma_{t,s,c}^{\text{max}} \quad \forall t \in \Omega^T, s \in \Omega^S, c \in \Omega^C \quad (10)$$

$$0 \leq q_{t,h}^{\text{dis}} \leq dis_h^{\text{max}} \cdot x_{t,h}^{\text{dis}}: \quad \phi_{t,h}^{\text{dmin}}, \phi_{t,h}^{\text{dmax}} \quad \forall t \in \Omega^T, h \in \Omega^H \quad (11)$$

$$0 \leq q_{t,h}^{\text{ch}} \leq ch_h^{\text{max}} \cdot x_{t,h}^{\text{ch}}: \quad \phi_{t,h}^{\text{cmin}}, \phi_{t,h}^{\text{cmax}} \quad \forall t \in \Omega^T, h \in \Omega^H \quad (12)$$

$$0 \leq k_{t,w} \leq k_{t,w}^{\text{max}}: \quad \eta_{t,w}^{\text{min}}, \eta_{t,w}^{\text{max}} \quad \forall t \in \Omega^T, w \in \Omega^W \quad (13)$$

$$-\pi \leq \theta_{t,s} \leq \pi: \quad \mu_{t,s}^{\text{max}}, \mu_{t,s}^{\text{min}} \quad \forall t \in \Omega^T, s \in \Omega^S \quad (14)$$

$$\theta_{t,s_1} = 0: \quad \mu_t^{\text{ref}} \quad \forall t \in \Omega^T \quad (15)$$

Objective function (1) maximizes the profit of the VSP. LMP at the bus to which a BSU is connected is equal to the negative of $\lambda_{t,s}$, the dual variable of the power balance Eq. (6) calculated endogenously in the lower level problem. BSU's profit comes from the difference of LMPs

when electricity is sold and purchased in the market. Therefore, the BSU's revenue is highly dependent on the LMP profile at the connecting bus.

The objective function (1) is subject to the upper-level constraints representing storage operation (2)–(4) and a lower-level problem simulating market clearing procedure (5)–(15).

Constraint (2) determines the current state of energy of a BSU based on its value in the previous time period, as well as charging and discharging amounts and efficiencies during the current time period. This equation considers both charging and discharging efficiencies. Constraint (3) forbids simultaneous charging and discharging of battery storage. Constraint (4) limits the state of energy of a BSU.

The objective function of the lower-level problem (5) maximizes the social welfare, which is defined as the difference between the consumers benefits (cleared quantities times the bidding prices) and the overall cost of suppliers (cleared quantities times the offering prices). As in most economic studies in the literature, the dc linear approximation of the network is used to represent nodal power balance and transmission line capacity limits. Eq. (6) enforces nodal power balance. Power flows through the lines are calculated in (7) and limited in (8). Constraint (9) imposes generator offering block limits, while (10) limits demand bidding blocks. Constraints (11) and (12) limit storage offering/bidding blocks, while constraint (13) imposes the upper limit on available wind generation. Since wind farms are considered to offer at 0 \$/MWh, their offers do not appear in the objective function of the lower-level problem. Consequently, the model will strive to use as much free wind power as possible. Constraint (14) limits voltage angles for each node, and constraint (15) sets the reference bus.

The model derives hourly offering curve of the VSP and maximizes its profit in the day-ahead market. It assumes all generator offers and demand bids are known to the VSP. This information can be derived using historical data and procedure proposed in [50].

Since the lower-level problem is continuous and linear, it can be replaced by its KKT conditions [51], i.e. first-order necessary conditions for a solution to be optimal, resulting in the following MPEC:

$$\max \sum_{i \in \Omega^T} \sum_{h \in \Omega^H} \lambda_{t,s(h)} \cdot (q_{t,h}^{\text{dis}} - q_{t,h}^{\text{ch}}) \quad (16)$$

subject to:

$$\text{Upper level constraints: (2)–(4)} \quad (17)$$

KKT conditions:

$$-\lambda_{i,b}^G - \lambda_{t,s(i)} + \gamma_{t,i,b}^{\min} - \gamma_{t,i,b}^{\max} = 0 \quad \forall t \in \Omega^T, i \in \Omega^I, b \in \Omega^B \quad (18)$$

$$\lambda_{s,c}^D + \lambda_{t,s} + \sigma_{t,s,c}^{\min} - \sigma_{t,s,c}^{\max} = 0 \quad \forall t \in \Omega^T, s \in \Omega^S, c \in \Omega^C \quad (19)$$

$$-\lambda_h^{\text{dis}} - \lambda_{t,s(h)} + \phi_{t,h}^{\text{dmin}} - \phi_{t,h}^{\text{dmax}} = 0 \quad \forall t \in \Omega^T, h \in \Omega^H \quad (20)$$

$$\lambda_h^{\text{ch}} + \lambda_{t,s(h)} + \phi_{t,h}^{\text{cmin}} - \phi_{t,h}^{\text{cmax}} = 0 \quad \forall t \in \Omega^T, h \in \Omega^H \quad (21)$$

$$-\lambda_{t,s(l)} - \beta_{t,l} + \beta_{t,l}^{\min} - \beta_{t,l}^{\max} = 0 \quad \forall t \in \Omega^T, l \in \Omega^L \quad (22)$$

$$\beta_{t,l} \cdot \text{susi} - \mu_{t,l}^{\text{ref}} + \mu_{t,s}^{\min} - \mu_{t,s}^{\max} = 0 \quad \forall t \in \Omega^T, s \in \Omega^S, l \in \Omega^{S(L)} \quad (23)$$

$$-\sum_{w \in \Omega^{W(S)}} k_{t,w} - \sum_{h \in \Omega^H(S)} q_{t,h}^{\text{dis}} - \sum_{i \in \Omega^I(S)} \sum_{b \in \Omega^B} g_{t,i,b} + \sum_{l \in \Omega^L(S)} pf_{t,l}^- - \sum_{l \in \Omega^L(S)} pf_{t,l}^+ = - \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} d_{t,s,c} - \sum_{h \in \Omega^H(S)} q_{t,h}^{\text{ch}} \quad \forall t \in \Omega^T \quad (24)$$

$$pf_{t,l} = \text{susi} \cdot (\theta_{t,s} - \theta_{t,m}) \quad \forall t \in \Omega^T, \{s, m\} \in \Omega^L, l \in \{s, m\} \quad (25)$$

$$\theta_{t,s_1} = 0 \quad \forall t \in \Omega^T \quad (26)$$

$$0 \leq pf_l^{\max} + pf_{t,l} \perp \beta_{t,l}^{\min} \geq 0 \quad \forall t \in \Omega^T, \forall l \in \Omega^L \quad (27)$$

$$0 \leq pf_l^{\max} - pf_{t,l} \perp \beta_{t,l}^{\max} \geq 0 \quad \forall t \in \Omega^T, \forall l \in \Omega^L \quad (28)$$

$$0 \leq g_{t,i,b} \perp \gamma_{t,i,b}^{\min} \geq 0 \quad \forall t \in \Omega^T, \forall i \in \Omega^I, \forall b \in \Omega^B \quad (29)$$

$$0 \leq g_{t,i,b}^{\max} - g_{t,i,b} \perp \gamma_{t,i,b}^{\max} \geq 0 \quad \forall t \in \Omega^T, \forall i \in \Omega^I, \forall b \in \Omega^B \quad (30)$$

$$0 \leq d_{t,s,c} \perp \sigma_{t,s,c}^{\min} \geq 0 \quad \forall t \in \Omega^T, \forall s \in \Omega^S, \forall c \in \Omega^C \quad (31)$$

$$0 \leq d_{t,s,c}^{\max} - d_{t,s,c} \perp \sigma_{t,s,c}^{\max} \geq 0 \quad \forall t \in \Omega^T, \forall s \in \Omega^S, \forall c \in \Omega^C \quad (32)$$

$$0 \leq q_{t,h}^{\text{dis}} \perp \phi_{t,h}^{\text{dmin}} \geq 0 \quad \forall t \in \Omega^T, \forall h \in \Omega^H \quad (33)$$

$$0 \leq \text{dis}_h^{\max} - q_{t,h}^{\text{dis}} \perp \phi_{t,h}^{\text{dmax}} \geq 0 \quad \forall t \in \Omega^T, \forall h \in \Omega^H \quad (34)$$

$$0 \leq q_{t,h}^{\text{ch}} \perp \phi_{t,h}^{\text{cmin}} \geq 0 \quad \forall t \in \Omega^T, \forall h \in \Omega^H \quad (35)$$

$$0 \leq \text{ch}_h^{\max} - q_{t,h}^{\text{ch}} \perp \phi_{t,h}^{\text{cmax}} \geq 0 \quad \forall t \in \Omega^T, \forall h \in \Omega^H \quad (36)$$

$$0 \leq k_{t,w} \perp \eta_{t,w}^{\min} \geq 0 \quad \forall t \in \Omega^T, \forall w \in \Omega^W \quad (37)$$

$$0 \leq k_{t,w}^{\max} - k_{t,w} \perp \eta_{t,w}^{\max} \geq 0 \quad \forall t \in \Omega^T, \forall w \in \Omega^W \quad (38)$$

$$0 \leq \pi + \theta_{t,s} \perp \mu_{t,s}^{\min} \geq 0 \quad \forall t \in \Omega^T, \forall s \in \Omega^S \quad (39)$$

$$0 \leq \pi - \theta_{t,s} \perp \mu_{t,s}^{\max} \geq 0 \quad \forall t \in \Omega^T, \forall s \in \Omega^S \quad (40)$$

This MPEC model (16)–(40) contains the following non-linearities:

1. the multiplication of dual variable $\lambda_{t,s}$ and primal variables $(q_{t,h}^{\text{dis}} - q_{t,h}^{\text{ch}})$ in the objective function;
2. the complementarity conditions (27)–(40).

To linearize the objective function, we use some of the KKT conditions. Using (20) and (21), the objective function is transformed to:

$$\max - \sum_{i \in \Omega^I} \sum_{h \in \Omega^H} (-\lambda_h^{\text{dis}} + \phi_{t,h}^{\text{dmin}} - \phi_{t,h}^{\text{dmax}}) \cdot q_{t,h}^{\text{dis}} + \sum_{i \in \Omega^I} \sum_{h \in \Omega^H} (-\lambda_h^{\text{ch}} - \phi_{t,h}^{\text{cmin}} + \phi_{t,h}^{\text{cmax}}) \cdot q_{t,h}^{\text{ch}} \quad (41)$$

From complementarity conditions (33)–(36), $\phi_{t,h}^{\text{dmin}} \cdot q_{t,h}^{\text{dis}} = 0$, $\phi_{t,h}^{\text{cmin}} \cdot q_{t,h}^{\text{ch}} = 0$, $\phi_{t,h}^{\text{dmax}} \cdot q_{t,h}^{\text{dis}} = \phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\max}$ and $\phi_{t,h}^{\text{cmax}} \cdot q_{t,h}^{\text{ch}} = \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\max}$ the following holds:

$$\lambda_{t,s(h)} \cdot (q_{t,h}^{\text{dis}} - q_{t,h}^{\text{ch}}) = \lambda_h^{\text{dis}} \cdot q_{t,h}^{\text{dis}} + \phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\max} - \lambda_h^{\text{ch}} \cdot q_{t,h}^{\text{ch}} + \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\max} \quad (42)$$

The complementarity conditions (27)–(40) are linearized using the well-known linear expressions from [52], where a general complementarity condition:

$$0 \leq x \perp y \geq 0 \quad (43)$$

is replaced by a set of linear constraints:

$$0 \leq x \leq i \cdot M \quad (44)$$

$$0 \leq y \leq (1-i) \cdot M \quad (45)$$

where M is a large enough constant and i is a binary variable. One should be careful when selecting the value for M . Infeasibility problems may occur if this value is too small, while M being too large increases the computational time.

The final problem is:

$$\max \lambda_h^{\text{dis}} \cdot q_{t,h}^{\text{dis}} + \phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\max} - \lambda_h^{\text{ch}} \cdot q_{t,h}^{\text{ch}} + \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\max} \quad (46)$$

subject to:

$$(17)–(25), (30)–(40), (26) \text{ and linearized variants of } (((27)–(29), (40))).$$

2.2. Price-quantity model formulation

The optimization problem that aims to maximize the profit of a VSP consisting of h BSUs and determine the bidding and offering prices as

well is stated as follows:

Refer Eq. (1)

subject to:

$$\alpha_{t,h}^{\text{dis}} \geq 0 \quad \forall t \in \Omega^T, h \in \Omega^H \quad (48)$$

$$\alpha_{t,h}^{\text{ch}} \geq 0 \quad \forall t \in \Omega^T, h \in \Omega^H \quad (49)$$

$$\text{Refer Eqs. (2)–(15)} \quad (50)$$

In lower-level objective function (5) the VSP now bids with unknown prices. Eqs. (48) and (49) enforce VSP's positive bidding and offering prices.

MPEC transformation is the same as in the quantity-only model with added (48) and (49) in the upper level. Also, (20) and (21) change, since known parameters λ_h^{ch} and λ_h^{dis} become unknown variables $\alpha_{t,h}^{\text{ch}}$ and $\alpha_{t,h}^{\text{dis}}$.

$$-\alpha_{t,h}^{\text{dis}} \lambda_{t,s(h)} + \phi_{t,h}^{\text{dmin}} - \phi_{t,h}^{\text{dmax}} = 0 \quad \forall t \in \Omega^T, h \in \Omega^H \quad (51)$$

$$\alpha_{t,h}^{\text{ch}} + \lambda_{t,s(h)} + \phi_{t,h}^{\text{cmin}} - \phi_{t,h}^{\text{cmax}} = 0 \quad \forall t \in \Omega^T, h \in \Omega^H \quad (52)$$

Using (51) and (52) for linearization, the objective function becomes:

$$\begin{aligned} \max - \sum_{t \in \Omega^T} \sum_{h \in \Omega^H} (-\alpha_{t,h}^{\text{dis}} + \phi_{t,h}^{\text{dmin}} - \phi_{t,h}^{\text{dmax}}) \cdot q_{t,h}^{\text{dis}} + \sum_{t \in \Omega^T} \sum_{h \in \Omega^H} (-\alpha_{t,h}^{\text{ch}} - \phi_{t,h}^{\text{cmin}} \\ + \phi_{t,h}^{\text{cmax}}) \cdot q_{t,h}^{\text{ch}} \end{aligned} \quad (53)$$

From complementarity conditions (33)–(36), $\phi_{t,h}^{\text{dmin}} \cdot q_{t,h}^{\text{dis}} = 0$, $\phi_{t,h}^{\text{cmin}} \cdot q_{t,h}^{\text{ch}} = 0$, $\phi_{t,h}^{\text{dmax}} \cdot q_{t,h}^{\text{dis}} = \phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\text{max}}$ and $\phi_{t,h}^{\text{cmax}} \cdot q_{t,h}^{\text{ch}} = \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\text{max}}$ the following holds:

$$\lambda_{t,s(h)} \cdot (q_{t,h}^{\text{dis}} - q_{t,h}^{\text{ch}}) = \alpha_{t,h}^{\text{dis}} \cdot q_{t,h}^{\text{dis}} + \phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\text{max}} - \alpha_{t,h}^{\text{ch}} \cdot q_{t,h}^{\text{ch}} + \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\text{max}} \quad (54)$$

The resulting objective function (54) still contains nonlinear multiplications of optimal offering/bidding prices and their quantities. Strong duality theorem states that primal and dual objective functions have the same values at the optimum. The strong duality equation is formulated as follows:

$$\begin{aligned} \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} \lambda_{s,c}^D \cdot d_{t,s,c} + \sum_{h \in \Omega^H} \alpha_{t,h}^{\text{ch}} \cdot q_{t,h}^{\text{ch}} - \sum_{i \in \Omega^I} \sum_{b \in \Omega^B} \lambda_{i,b}^G \cdot g_{i,b} \\ - \sum_{h \in \Omega^H} \alpha_{t,h}^{\text{dis}} \cdot q_{t,h}^{\text{dis}} = \sum_{l \in \Omega^L} (\beta_{l,l}^{\text{min}} \cdot \text{pf}_l^{\text{max}} + \beta_{l,l}^{\text{min}} \cdot \text{pf}_l^{\text{max}}) \\ + \sum_{i \in \Omega^I} \sum_{b \in \Omega^B} \gamma_{i,b}^{\text{max}} \cdot g_{i,b}^{\text{max}} + \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} \sigma_{t,s,c}^{\text{max}} \cdot d_{t,s,c}^{\text{max}} \\ + \sum_{h \in \Omega^H} (\phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\text{max}} + \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\text{max}}) \\ + \sum_{w \in \Omega^W} \eta^{\text{max}} \cdot k^{\text{max}} \\ + \sum_{s \in \Omega^S} (\pi \cdot \mu_{t,s}^{\text{min}} + \pi \cdot \mu_{t,s}^{\text{max}}) \quad \forall t \in \Omega^T \end{aligned} \quad (55)$$

From Eq. (55), the nonlinear terms can be easily expressed as linear:

$$\begin{aligned} \sum_{h \in \Omega^H} \alpha_{t,h}^{\text{dis}} \cdot q_{t,h}^{\text{dis}} - \sum_{h \in \Omega^H} \alpha_{t,h}^{\text{ch}} \cdot q_{t,h}^{\text{ch}} = - \sum_{l \in \Omega^L} (\beta_{l,l}^{\text{min}} \cdot \text{pf}_l^{\text{max}} + \beta_{l,l}^{\text{min}} \cdot \text{pf}_l^{\text{max}}) \\ - \sum_{i \in \Omega^I} \sum_{b \in \Omega^B} \gamma_{i,b}^{\text{max}} \cdot g_{i,b}^{\text{max}} - \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} \sigma_{t,s,c}^{\text{max}} \cdot d_{t,s,c}^{\text{max}} \\ - \sum_{h \in \Omega^H} (\phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\text{max}} - \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\text{max}}) \\ - \sum_{w \in \Omega^W} \eta^{\text{max}} \cdot k^{\text{max}} - \sum_{s \in \Omega^S} (\pi \cdot \mu_{t,s}^{\text{min}} + \pi \cdot \mu_{t,s}^{\text{max}}) \\ + \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} \lambda_{s,c}^D \cdot d_{t,s,c} - \sum_{i \in \Omega^I} \sum_{b \in \Omega^B} \lambda_{i,b}^G \cdot g_{i,b} \end{aligned} \quad (56)$$

The final objective function is:

$$\begin{aligned} \max \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} \lambda_{s,c}^D \cdot d_{t,s,c} - \sum_{i \in \Omega^I} \sum_{b \in \Omega^B} \lambda_{i,b}^G \cdot g_{i,b} \\ - \sum_{l \in \Omega^L} (\beta_{l,l}^{\text{min}} \cdot \text{pf}_l^{\text{max}} + \beta_{l,l}^{\text{min}} \cdot \text{pf}_l^{\text{max}}) - \sum_{i \in \Omega^I} \sum_{b \in \Omega^B} \gamma_{i,b}^{\text{max}} \cdot g_{i,b}^{\text{max}} \\ - \sum_{s \in \Omega^S} \sum_{c \in \Omega^C} \sigma_{t,s,c}^{\text{max}} \cdot d_{t,s,c}^{\text{max}} - \sum_{h \in \Omega^H} (\phi_{t,h}^{\text{dmax}} \cdot \text{dis}_h^{\text{max}} + \phi_{t,h}^{\text{cmax}} \cdot \text{ch}_h^{\text{max}}) \\ - \sum_{w \in \Omega^W} \eta^{\text{max}} \cdot k^{\text{max}} - \sum_{s \in \Omega^S} (\pi \cdot \mu_{t,s}^{\text{min}} + \pi \cdot \mu_{t,s}^{\text{max}}) \end{aligned} \quad (57)$$

2.3. EPEC formulation

The MPEC formulations from the previous sections assume that a

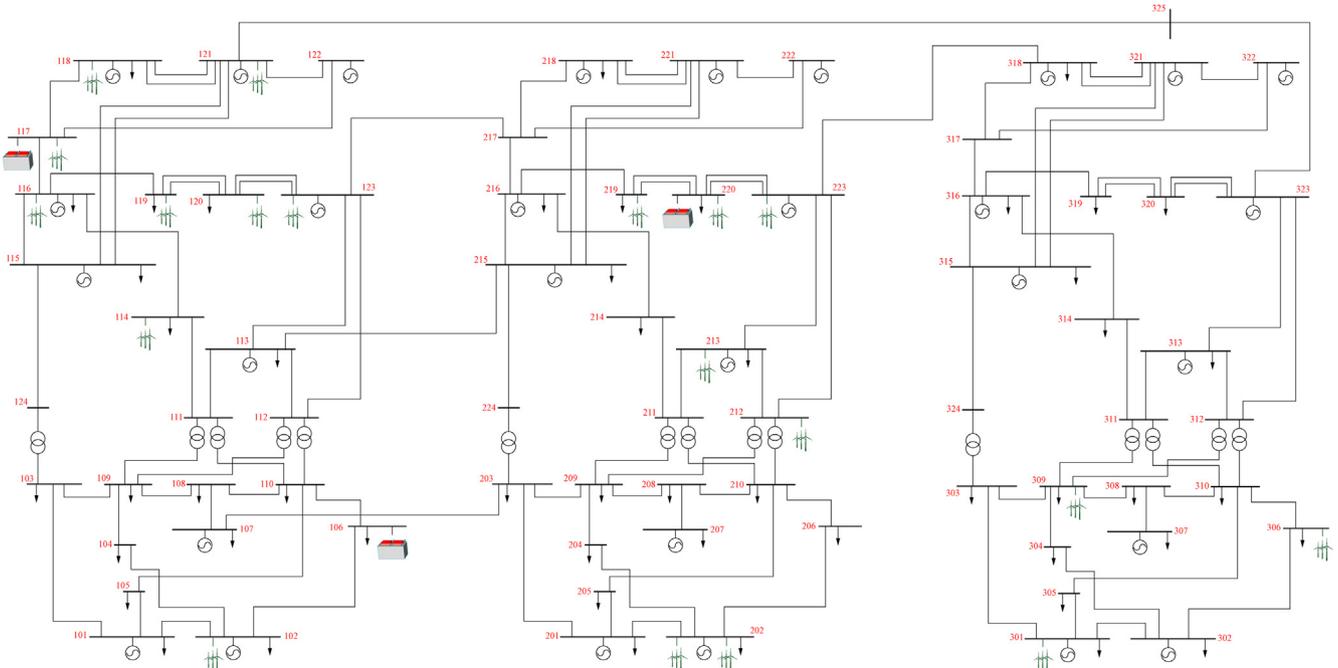


Fig. 1. IEEE RTS-96 test system with 19 wind farms and 3 BSUs.

single VSP owns all the BSUs in the system. However, one may expect multiple VSP owners operating their BSUs and competing for profit, which resembles an EPEC structure. The solution of this EPEC problem is a set of equilibria, in which none of the VSPs is able to increase its revenue unilaterally by changing the offering/bidding quantities of its BSUs. To solve the proposed EPEC we use the diagonalization algorithm, which is implemented by sequentially solving one MPEC at a time. That is, MPECs are solved one by one, considering fixed the decisions of the remaining MPECs. Once all VSP's MPECs are solved, the solving cycle restarts as many times as needed for the decision variables of each MPEC to stabilize. Additional information on the use and properties of the diagonalization procedure are available in [53].

The upper-level problem schedules the BSUs to maximize the profit of the current VSP, while the lower-level problem considers both the current VSP and all other VSPs with their decisions fixed from the previous iteration. In the price-quantity environment, every BSU has an incentive to lower the offering price in order to seize the market. The price lowering continues until reaching the marginal costs. Therefore, we use the quantity-only model where BSUs offer at zero price and bid at market cap price.

To make distinction between BSU ownership, a new set Ω^j is added. Objective function (46) and upper-level constraints (17) are valid $\forall h \in \Omega^{H(j)}$, i.e., for all h pertaining to owner j . The equations from the lower-level problem consider all the BSUs participating in the market, i.e., they constrain all $h \in \Omega^H$.

3. Case study

The proposed model is tested on IEEE RTS-96 supplemented with 19 wind farms (see Fig. 1), whose output throughout the representative week is shown in Fig. 2. The representative week is obtained from the annual data available in [54] using the fast-forward scenario reduction algorithm described in [55]. During this week, the overall system load ranges from 3900 MW to 8100 MW, while the daily consumption ranges from 121 GWh to 161 GWh. Available wind power ranges from 1 to 3% in some hours of days 2 and 5, and all the way to over 100% of the load during the night hours between days 1–2 and 6–7. Three BSUs are located at buses 106, 117 and 220, which are identified as attractive locations for performing arbitrage [20]. Their energy capacity in this case study is 100, 250 or 500 MWh each, which is almost negligible in terms of daily consumption during the low available wind power, but can be significant during the days in which the available wind power is comparable to the overall load.

The presented case study is implemented in GAMS 24.5 and solved using CPLEX 12.6. The optimality gap is set to 0,5%.

3.1. Effects of optimization time horizon

An important issue when optimizing a merchant BSU is the optimization time horizon. Here, we analyze if it is sufficient to consider only one day at a time or if a longer look-ahead period may bring higher profit. To examine this, we analyze profits of a single 250 MWh BSU located at bus 117. Table 1 compares its profits using one-day look-ahead, two-days look-ahead, and a week look-ahead scheduling horizon (all three cases consider day-ahead market where market bids and offers are submitted one day in advance). The results show that the loss of profit when looking only a day ahead, without trying to forecast the prices after this day, is 32% as opposed to the case when the prices are accurately forecasted for one week ahead. On the other hand, the two-day look-ahead horizon performs as well as the one week look-ahead horizon. This is because the duration of storage is one hour, so there is no long-term storing of energy. This indicates that a merchant-owned BSU should forecast market outcomes for two days in advance, and not only for the following day. Longer scheduling horizon allows a BSU to precharge and/or preserve the stored energy from the previous day, thus gaining higher overall profit. However, although a BSU

significantly benefits from longer scheduling horizons, it is difficult to accurately predict load levels and market clearing outcomes a week ahead. On the other hand, the two day look-ahead scheduling provides a good balance between uncertainty related to market outcomes and optimal charging/discharging decisions. For this reason, in the remaining subsections of the case study we use a 48-h ahead scheduling, apply the results for the first 24 h, which is the day-ahead market horizon, and discard the last 24 h. After that, we move to the next day and perform optimization for the second and the third day, discarding the results for the third day, and so on.

3.2. Impact of BSU capacity on revenue

Fig. 3 shows charging/discharging schedules, states of energy and LMP profiles of three BSUs with 100 MW capacity consisting a VSP. At the beginning of the week the BSUs were empty, while their states of energy transfer from one day to another is based on the state of energy at hour 24 of each optimization. The BSUs are connected to buses 106, 117 and 220 and their charging and discharging capacity is set to 1C, i.e. they can fully charge or discharge within one hour.

During the first five days all three BSUs behave in a similar way performing one full charging/discharging cycle a day. The only exception is the BSU at bus 117, which performs an additional cycle right at the beginning of the first day. Generally, all the BSUs charge during the low-price periods and discharge during the high-price periods. However, the charging and discharging volumes are rarely 100 MW, which indicates that higher volumes would have negative impact on LMPs. On day 6, the BSUs at buses 106 and 117 are idle as they can not take advantage of the volatile prices. Namely, the LMPs decrease throughout the day 6 and there is no opportunity for performing a charging/discharging cycle for these two BSUs. On the other hand, firm and more volatile LMPs at bus 220 allow this BSU one half and one full cycle during day 6. The last day is abundant with wind power and LMPs are extremely low. Regardless, the BSUs at buses 106 and 220 manage to perform two full cycles, while the BSU at bus 117 performs a single cycle. The results in Fig. 3 also indicate that the BSUs have different state of energy at the end of each day. For instance, the BSU at bus 106 is fully charged at the end of the first day and fully discharged at the end of the second day. This confirms the importance of the two day look-ahead optimization horizon.

Daily profit of each BSU is provided in Table 2. The highest overall profit is obtained for the BSU at bus 220. However, it is interesting to analyze the daily distribution of the profits. The most profitable days are days 2 and 4. Day 2 is profitable because the BSUs fully charge in the last hour of the first day and manage to discharge at high prices

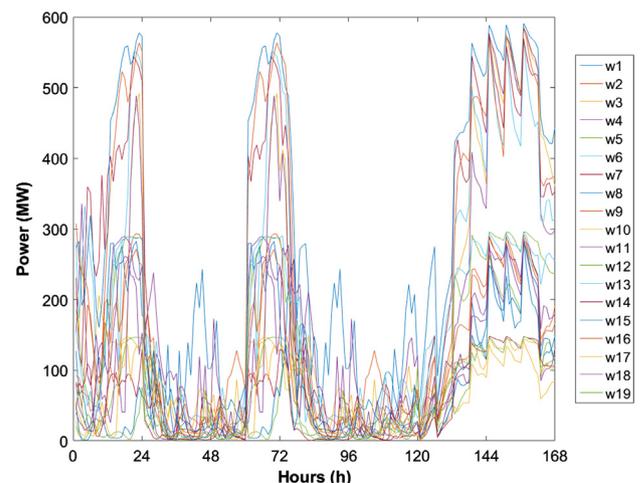


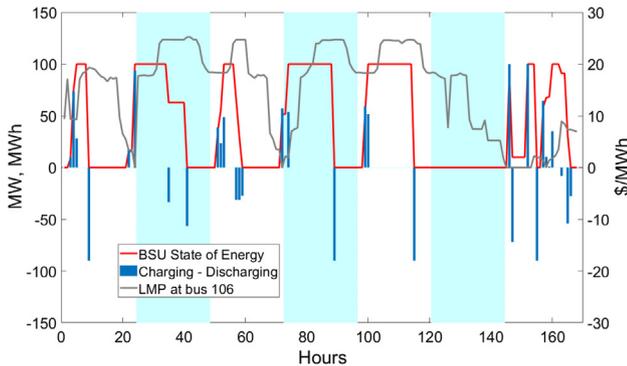
Fig. 2. Available output of 19 wind farms within the IEEE-RTS 96 during the representative week [54].

Table 1
Storage profit depending on the optimization horizon, \$.

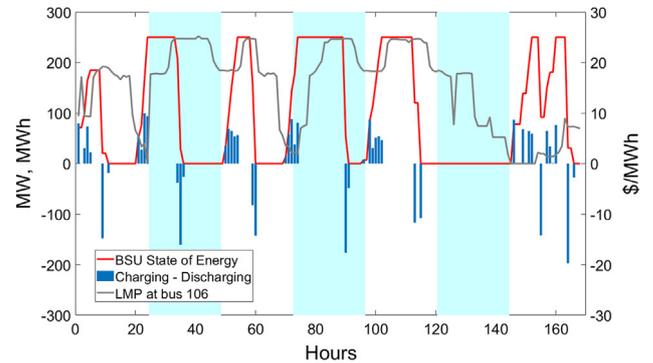
	Day ahead	Two days ahead	Week ahead
Day 1	1266	1209	1209
Day 2	680	5619	5619
Day 3	470	470	470
Day 4	5567	5567	5568
Day 5	486	481	482
Day 6	486	587	597
Day 7	1644	1642	1642
Total	10,600	15,576	15,587

Table 2
Daily individual BSU profits for 100 MWh capacity, \$ (%).

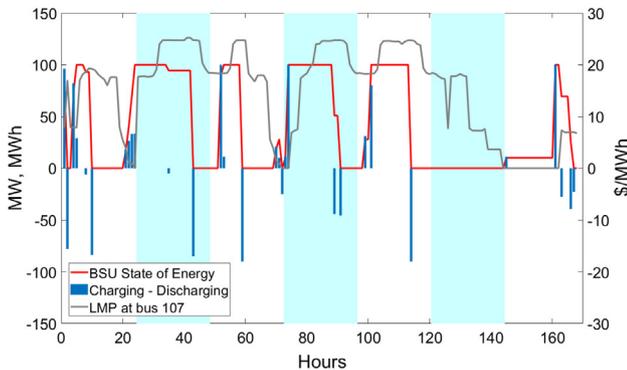
	Bus 106	Bus 117	Bus 220	Total
Day 1	641	1287	386	2315
Day 2	2255	2228	2250	6733
Day 3	188	150	188	526
Day 4	2128	2227	2227	6852
Day 5	188	194	191	573
Day 6	0	0	688	688
Day 7	694	639	1044	2377
Total	6094	6725	6974	19,794



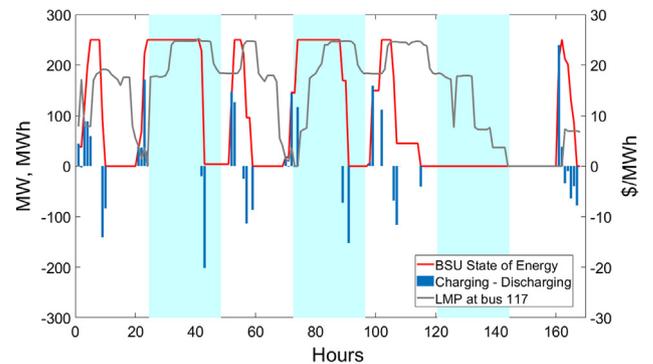
(a) BSU at bus 106



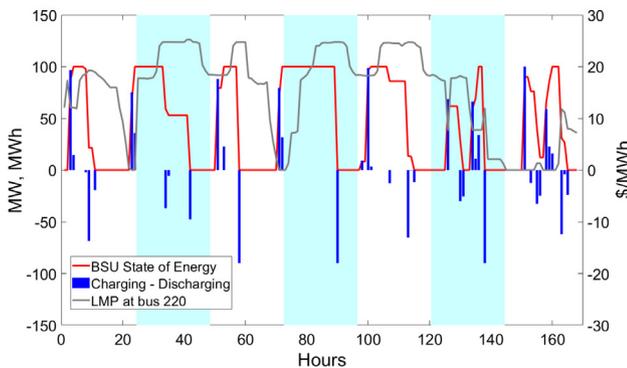
(a) BSU at bus 106



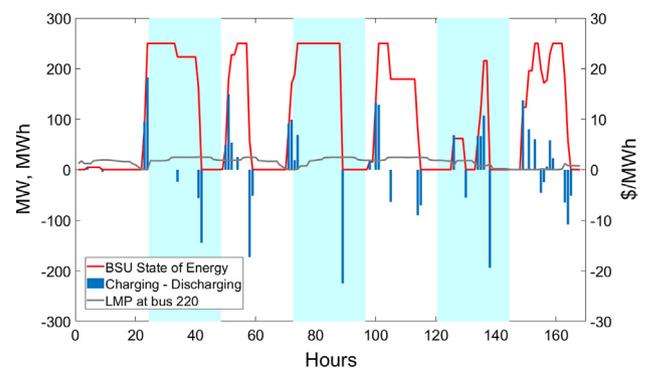
(b) BSU at bus 117



(b) BSU at bus 117



(c) BSU at bus 220



(c) BSU at bus 220

Fig. 3. VSP charging/discharging schedules, states of energy and LMP profiles for 100 MWh BSU capacity.

during the second day (around \$25). The fourth day is also profitable because the BSUs are charged at the end of day 3 and beginning of day 4 at zero price and then discharged in the afternoon of day 4 at around \$25. In day 1, the BSU at bus 117 is most profitable due to an additional

Fig. 4. VSP charging/discharging schedules, states of energy and LMP profiles for 250 MWh BSU capacity.

cycle in the first few hours. Despite the similar state of energy curve during day 1, the BSU at bus 220 is less profitable than the one at bus 106 because it charges at higher cost.

Fig. 4 shows charging/discharging schedules, states of energy and

LMP profiles for 250 MWh capacity BSUs. The charging/discharging schedules are similar to the ones for 100 MWh capacity BSUs, with some slight differences. The BSU at bus 106 is not fully charged during the first day in order to preserve the fairly low LMPs. Also, since it requires more energy to fully charge at the end of the first day, it partly charges at higher prices, thus resulting in lower daily profit (compare in Tables 2 and 3). Also, it performs only two cycles during day 7, but the daily profit is increased due to higher energy volume traded in the market. The schedule of the BSU at bus 117 does not change much, but one can note a small negative profit in day 3. This is a result of the charging process at the end of the day needed to achieve high profit in day 4. The 250 MWh BSU at bus 220 is scheduled with an extremely shallow cycle during the first day, as opposed to the full cycle for the 100 MWh BSU capacity. This is the result of congestion between the BSUs at buses 117 and 220. Higher charging quantity of the BSU at bus 220 would incur higher LMP at bus 117. This situation clearly depicts the joint coordination of the three BSUs within a VSP with the goal of maximizing profit for the entire VSP and not a single BSU. It is also worth noting that, as opposed to the 100 MWh BSUs, the 250 MWh BSUs never charge nor discharge at maximum rate since this would cause undesired changes in LMPs.

Profit of the BSU at bus 117 in Table 1 is higher than the one in Table 3 because the BSUs at buses 106 and 220 are not considered in Table 1. This indicates that existence of additional BSUs reduced the value of a BSU in the system.

BSU charging/discharging schedules, states of energy and LMP profiles for 500 MWh capacity BSUs are shown in Fig. 5. As opposed to the 100 MWh and 250 MWh cases, the BSU at bus 106 does not perform a charging/discharging cycle during the first day. Instead, it charges to full capacity and preserves the energy for the second day, when the prices are much higher. The result of this schedule is negative profit of this BSU on Day 1, which is followed by an \$8,994 profit in the second day, as shown in Table 4. The BSU connected to bus 117 performs a reduced cycle, i.e. it does not fully charge, during the first day, but the remaining days of the week follow similar schedules as in case of 100 MWh and 250 MWh BSU capacities. The BSU connected to bus 220 also is also scheduled very similar as in the 250 MWh case, but with reduced charging cycle during the day 6. Table 4 indicates huge differences in profits for individual days. The most profitable days for the VSP are days 2 and 4. This is a direct outcome of the wind profile in Fig. 2, where day 1 is rich with wind energy, which is charged to the VSP and injected into the system during day 2, which has very low wind output and, consequently, high LMPs. Similarly, the late hours of day 3 and early hours of day 4 are abundant in wind output, which is stored in the VSP and discharged in the second half of day 4 at high prices. During day 6, the BSUs at buses 106 and 117 are idle and their profit in that day is zero. This is a direct result of the increasing wind throughout the day, which results in almost monotonically reducing LMPs throughout the day. Since the final LMPs in day 6 at most buses is zero, there are no arbitrage opportunities for the BSUs at buses 106 and 117.

A comparison of the total VSP profit for different BSU capacities (Tables 2–4) indicates the saturation of profit as the BSU capacity increases. Specifically, the overall VSP profit for 250 MWh installed BSU capacity is 2.17 times higher than in the case of 100 MWh capacity, while the overall profit for 500 MWh BSUs is only 3.84 times higher than in the case of 100 MWh BSUs.

3.3. Analysis of VSP offering and bidding prices

When a BSU is charging, it is adding up to the total system load. As a result, its purchase bid may drive the LMPs up resulting in higher purchasing price of electricity. Similarly, when discharging, BSU acts as a generation resource and may reduce the LMP, resulting in a lower selling price. For this reason, the BSU offering and bidding prices, i.e. variables $\alpha_{t,h}^{\text{dis}}$ and $\alpha_{t,h}^{\text{ch}}$ from the model presented in Section 2.3, are for the most part identical to the expected LMPs.

Table 3

Daily individual BSU profits for 250 MWh capacity, \$ (%).

	Bus 106	Bus 117	Bus 220	Total
Day 1	459	1309	18	1786
Day 2	5568	5480	5596	16,644
Day 3	7	–28	463	442
Day 4	5189	5566	5473	16,228
Day 5	603	456	467	1527
Day 6	0	0	923	923
Day 7	1648	1582	2134	5364
Total	13,474	14,366	15,075	42,914

To maximize its profit, VSP bids and offers quantities that do not alter the LMPs significantly. In Figs. 6–8, the VSP charging bids are marked with red circles and discharging offers with blue circles. All the bids are accepted and the circles basically represent the time periods in which a BSU was charged (red circles) or discharged (blue circles).

The LMPs with and without BSUs in Figs. 6–8 indicate very low changes in LMPs due to BSU actions. In most graphs, the blue line, representing the LMPs when there are no BSUs in the system, is behind the red line, which shows the LMPs when BSUs are participating in the market. The increased LMPs appear around hour 72 for 100 and 250 MWh capacities of the BSU at bus 106 (Figs. 6a and b). The 250 MWh BSU at bus 117 even manages to perform a small charging/discharging cycle around hour 72 (Fig. 7b). However, an interesting situation occurs at the end of the third day for the 100 MWh BSU at bus 117. This BSU actually discharges at zero price and then charges in the next two hours, again at zero price. The discharged quantity is actually quite high, around 25 MWh (see Fig. 3b). Although this small charging/discharging cycle has no effect on the objective function, since both the charging and discharging prices are zero, this should be avoided as it unnecessary increases degradation of the BSU. This can be avoided by implementing a degradation model, e.g. [56]. A reduction of the LMPs due to BSU discharging is noticed at the end of the last day, e.g. Figs. 6b, 8a and 8b.

3.4. Competition between the VSPs using EPEC

In the previous subsections of the case study, all three BSUs were owned and operated by a single VSP, which means they offer and bid in the market in a coordinated manner. Here, we compare the results of the coordinated scheduling of the three 100 MWh BSUs under a single VSP (Table 2) to a setting in which each of the BSUs has a different owner. In this case, all three BSUs, now each of them being a VSP of its own, maximize their profit independently of the other two VSPs and compete among each other.

The competitive price setting optimization model is a classical Bertrand model [57]. In a Bertrand pricing game, a Nash equilibrium¹ is found when all competitors bid at the same price, which is equal to the marginal cost. If the price set by the competitors is the same but higher than the marginal cost, there will be an incentive for the competitors to lower their prices and seize the market. Therefore, the only equilibrium in which none of the competitors will be willing to deviate is when the price equals competitors' marginal cost. When optimized simultaneously, BSUs offer at a price that is usually higher than their marginal cost which is assumed to be zero. In the EPEC model, offering at a price above marginal cost would leave a competitor VSP an incentive to lower its offering price to seize the market. The process of lowering the price to seize the market would repeat until all the VSPs offer at their marginal cost.

¹ Nash equilibrium is a game theory solution concept of a non-cooperative game that involves several players. Nash equilibrium is a point in which any change to a player's strategy does not result in additional benefit. Detailed information are available in [58].

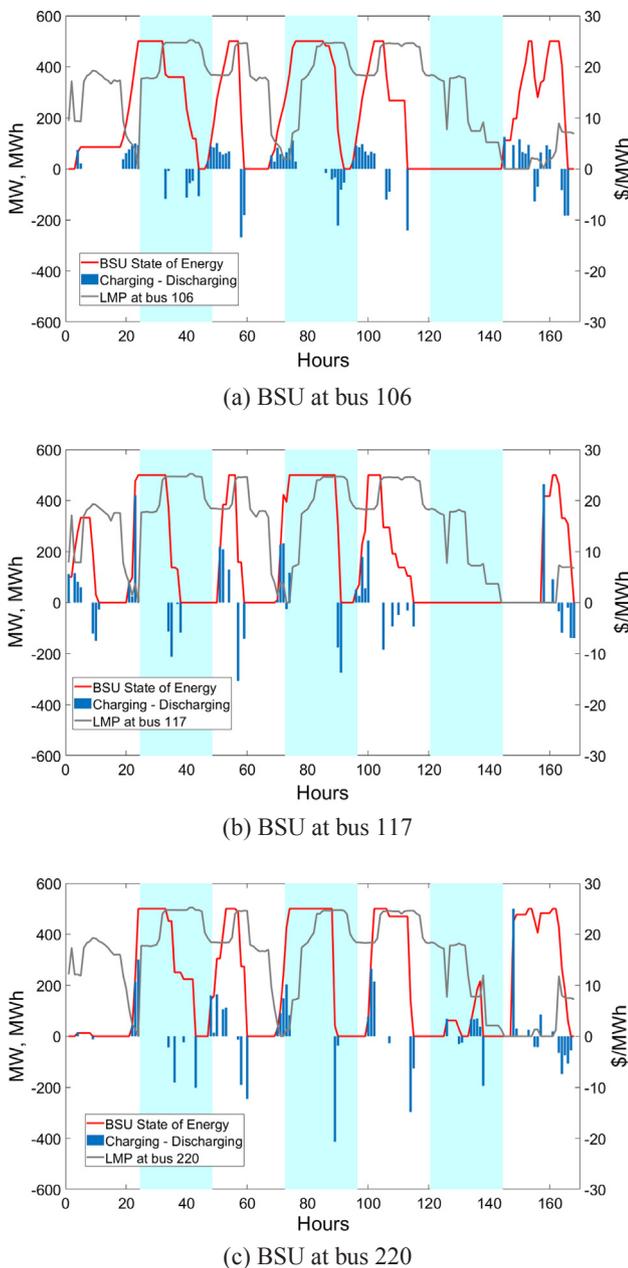


Fig. 5. VSP charging/discharging schedules, states of energy and LMP profiles for 500 MWh BSU capacity.

Table 4
Daily individual BSU profits for 500 MWh capacity, \$ (%).

	Bus 106	Bus 117	Bus 220	Total
Day 1	-2891	857	-346	-2380
Day 2	8994	11,133	8179	28,306
Day 3	1764	-855	3725	4633
Day 4	7546	10,179	10,756	28,480
Day 5	3150	1801	886	5837
Day 6	0	0	932	932
Day 7	3229	3118	3840	10,187
Total	31,792	26,233	27,970	75,995

Since the BSUs' marginal operating cost is zero, in EPEC model we use the quantity-only offering model from [59] with offering price set to \$0 and bidding price to the market cap. The diagonalization algorithm used to solve the EPEC is implemented by sequentially solving an MPEC

for each VSP considering fixed the decisions of the remaining VSPs. When using the diagonalization algorithm, the profit that a BSU (VSP) makes greatly depends on its precedence to set the offering/bidding quantities. The aim of this analysis is to characterize possible equilibria. It is important to emphasize that the presented diagonalization procedure does not reflect actual mechanics of the VSP competition, as all VSPs submit their offer simultaneously. Instead, this analysis aims at characterizing possible equilibria that can occur in this competition and find a range of possible VSP profits when competing with other VSPs.

Profits of individual 100 MWh VSPs for all six possible equilibrium outcomes are listed in Table 5. For example, if the VSP connected to bus 106 is scheduled first, its profit is \$6705 or \$6802, depending on the scheduling sequence of the other two VSPs. These two profits are higher than the profits where the BSU at bus 106 is a part of the bigger VSP (\$6094), but the profit of the other two VSPs are decreased. The overall profit of the three VSPs is \$18,910 and \$18,992, depending on the scheduling sequence of the other two VSPs, which is over 4% lower than in case of the coordinated approach of all three BSUs under a single VSP (see Table 2). Similar conclusions are derived for the other two VSPs. Regardless of the sequence of the VSP scheduling, their overall profit is lower (up to 7%) than if their actions are coordinated under a single VSP. Considering that all market participants submit their offers and bids individually and independently and that the market outcome is known only after the market operator performs the clearing procedure, it is important to understand the meaning of the profits listed in Table 5. The profits where a specific VSP solves its MPEC first in the EPEC procedure represent an upper bound on its possible profit in the market, while the lower profits indicate the lower bound of the possible VSP profit. The actual VSP profit depends on its quality of scheduling and accurate consideration of the other VSPs' decision-making processes.

3.5. Neglecting other VSPs at the bidding stage

In order to evaluate the importance of considering other VSPs at the bidding stage, we perform a simulation where each of the three VSPs (each VSP owning a single BSU) derives its optimal bidding strategy by completely neglecting other VSPs and their bidding strategies. This is achieved by using the MPEC from Section 2.2 and ignoring the BSUs owned by other VSPs. The obtained VSP bidding strategies are then used to simulate actual market clearing represented by constraints (5)–(15). The resulting LMPs, which may be different than those expected at the scheduling stage, are then used to determine the actual VSP profits.

Table 6 shows the reduction of profit as compared to Table 2. For 100 MWh VSPs, the overall weekly profit of the VSP at bus 106 reduces from \$6094 to only \$270. This is mainly because of a huge spike in LMP at this bus at hour 24 (see Fig. 9a) caused by neglecting the other VSPs in the system at the bidding stage. Actually, at hour 24 the LMP at all three VSP buses is zero and all three VSPs decided to charge at full capacity. This caused an extremely high LMP at bus 106 (over \$60) and caused great monetary losses to the VSP at bus 106. The VSP losses on day 2 are negligible, while on day 3 the VSP at bus 117 actually had higher profit due to reduced profit for the VSPs at buses 106 and 220 (the overall profit of all the VSPs on day 3 is reduced by 35%). The profit on days 4–6 is only slightly reduced, while the overall profit on day 7 is reduced by 26%. In total, the VSPs made 34% lower profit as compared to their coordinated bidding presented in Table 2. The results of the simulations indicate that with the increasing energy storage capacity in the power system, it is necessary to anticipate the competitors' decisions. This might be harder task than anticipating the strategic decisions of the generators as the BSUs do not have (or have very low) operating costs and throughout the day may bid different quantities at different prices. The generators are making profit as long as the LMP is higher than their marginal cost. On the other hand, BSUs are making profit if their selling price is higher than their purchasing price plus the

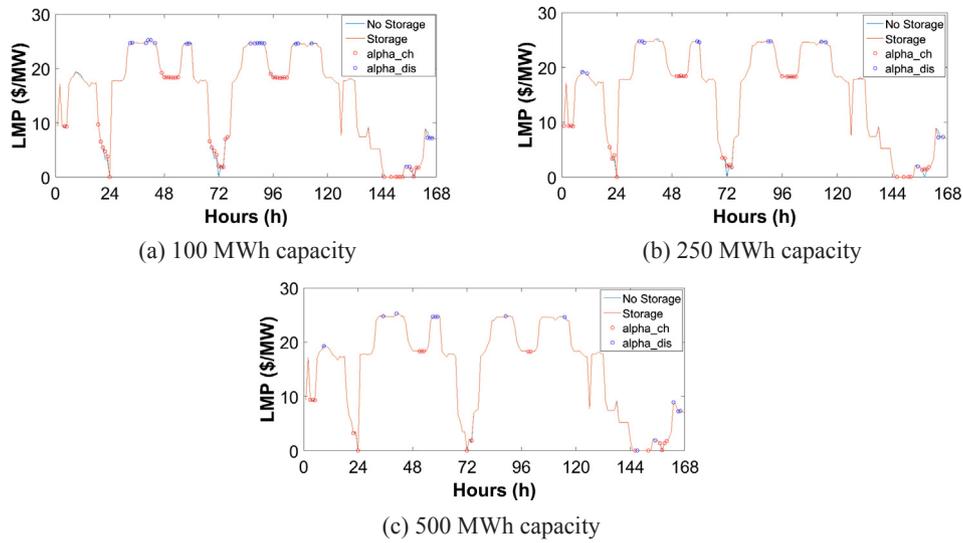


Fig. 6. BSU at bus 106 offering strategy and LMPs.

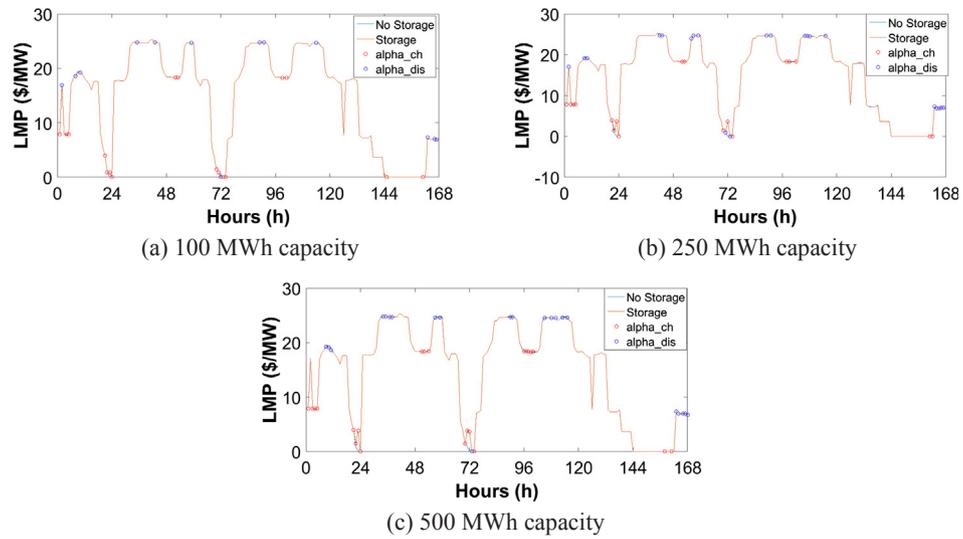


Fig. 7. BSU at bus 117 offering strategy and LMPs.

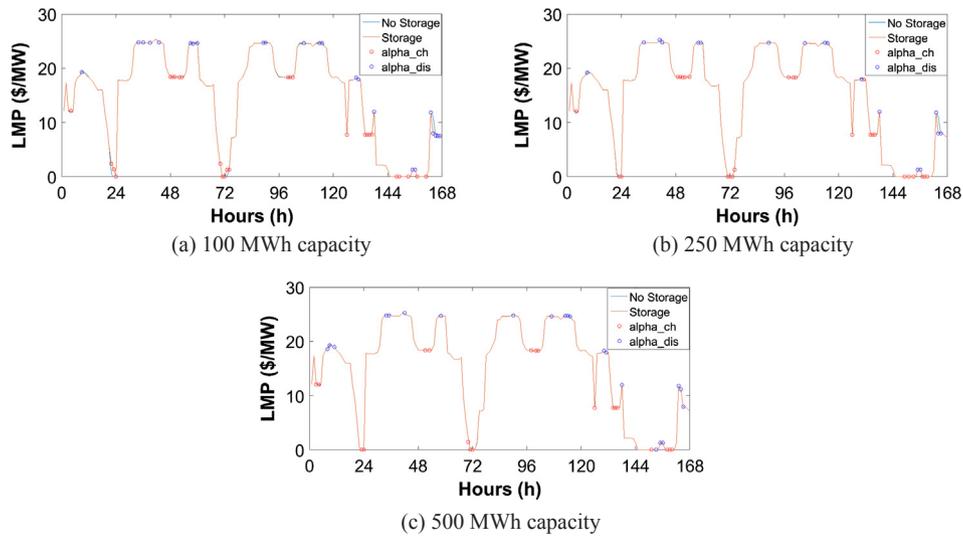


Fig. 8. BSU at bus 220 offering strategy and LMPs.

Table 5
Profit of 100 MWh VSPs at buses 106, 117 and 220 for different VSP scheduling sequences, \$.

First VSP	106		117		220	
	117	220	106	220	106	117
106	6705	6802	5688	5204	55,547	5002
117	6120	5829	7149	7212	5976	6120
220	6085	6361	6118	6409	7407	7292
Total	18,910	18,992	18,955	18,825	18,930	18,414

Table 6
Daily individual BSU profits for 100 MWh capacity when neglecting other VSPs at the scheduling stage, \$ (%).

	Bus 106	Bus 117	Bus 220	Total
Day 1	-4993 (-∞%)	1098 (-15%)	275 (-29%)	-3620 (-∞%)
Day 2	2228 (-1%)	2228 (0%)	2228 (-1%)	6683 (-1%)
Day 3	77 (-59%)	188 (26%)	79 (-58%)	344 (-35%)
Day 4	2128 (0%)	2227 (0%)	2227 (0%)	6581 (0%)
Day 5	188 (0%)	184 (-5%)	191 (0%)	564 (-2%)
Day 6	0 (0%)	0 (0%)	677 (-2%)	677 (-2%)
Day 7	642 (-8%)	487 (-24%)	635 (-39%)	1764 (-26%)
Total	270 (-96%)	6412 (-5%)	6311 (-10%)	12993 (-34%)

cycle efficiency. This means that the outcome of the quantity-only offering model with selling price set to zero and purchasing price at market cap can be significantly different than anticipated. Therefore, the VSPs should consider using the price-and-quantity offering model to protect against the undesired market outcomes.

4. Conclusions

This paper exploits MPEC and EPEC structure to evaluate the profit opportunities of BSUs in the day-ahead energy market. The following conclusions are derived:

1. Due to energy preservation and precharge abilities, a BSU should forecast market outcomes for two days in advance in order to

maximize its overall profits. However, forecasting market prices, especially as a price taker in a system with high integration of wind energy might be a difficult task and two-days scheduling horizon might not be optimal in for specific cases. Therefore, the quality of forecasting market prices will determine the look-ahead horizon for energy storage.

2. BSUs behave in a way to minimize their impact on LMPs. For this reason, they may discharge/charge at hours whose LMPs are not highest/lowest. As a consequence, in the analyzed case study they usually charge and discharge during multiple hours (despite the 1 h duration of storage) in order to not affect the LMPs and reduce their profits.
3. The characteristics of the BSUs from the case study are suitable for performing daily arbitrage. However, in order to maximize their profit, the BSUs might miss their daily charging/discharging cycle in order to charge at very low price and preserve the energy for discharging at high prices in the following day.
4. Comparison of the MPEC and EPEC settings allows BSUs to reach an equilibrium encompassing their individual maximum revenue targets.
5. Coordinated BSU strategy in the day-ahead market results in significantly higher profits as compared to the uncoordinated EPEC approach.
6. In the uncoordinated approach, BSU profits are highly dependent on the BSU scheduling sequence, which means that there are many equilibria with uneven distribution of profits.
7. Discarding other storage facilities at the day-ahead scheduling phase may result in a huge reduction of profit or even incur a loss for a VSP. This is because only a slightly higher charging/discharging level may cause severe upward/downward price spikes.

The running time for the 48-h horizon in all the simulations is below 2 min, which makes it useful for medium-scale power systems. Generator offering curves may be derived using the historical market data and inverse optimization techniques. However, the impact of uncertainty of wind generation and load levels remains to be investigated in future research. This would allow assessing the impact of forecasting errors and enable an additional revenue stream for the VSPs from the intraday and/or balancing markets.

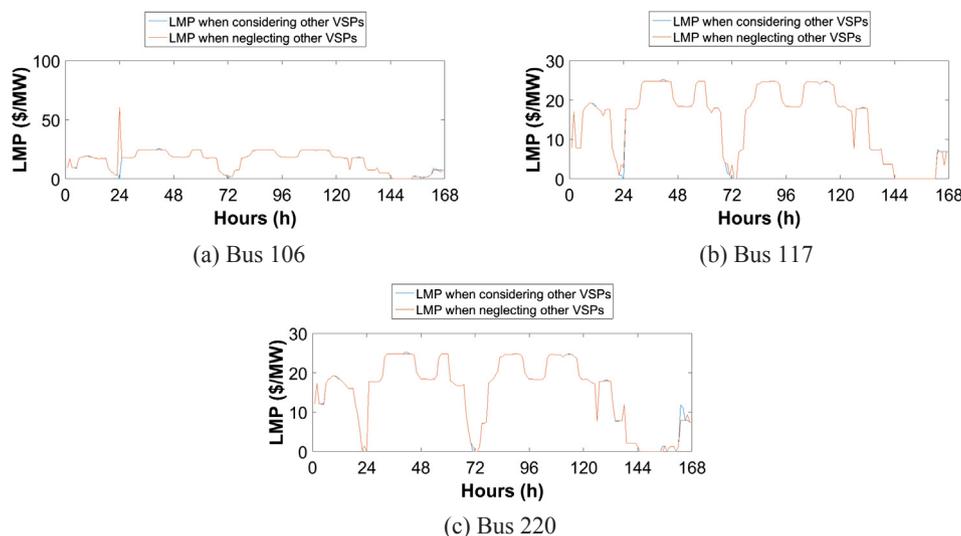


Fig. 9. Difference in LMPs after the actual market clearing when the VSPs neglect and when they consider other VSPs at the scheduling stage.

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